Tuning neutron rf-pulse frequency in the nEDM experiment

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Abstract

In the Paul Sherrer Institute in Villigen, Switzerland an experiment aims to measure neutron's electric dipole moment with use of Ramsey method of separated oscillating fields: two magnetic pulses are applied to polarized Ultra-Cold Neutrons with time gap in between to allow them to precess freely in constant magnetic and electric fields. A simple computer model of the experiment was created in order to investigate several methods of choosing pulses' frequency. The best method was determined, althogh all that were proposed showed equal efect on experiment's result uncertainty.

Note about notation

This paper often reffers to Larmor frequencies of neturons and mercury atoms, both being proportional to magnetic field value. For simplicity symbol $\mathfrak{f}$ (Gothic f) is introduced, defined as Larmor frequency divided by appropriate gyromagnetic ratio: $\mathfrak{f}_{\mathfrak{x}} := f_X/\gamma_X$.

nEDM experiment

Many Standard Model extensions predict very small, but non-zero value of neutron's electric dipole moment. Therefore determining its value provides a direct verification for these theories.

Unfortunately nEDM (common abbreviation for neutron's electric dipole moment) is incredibly small. Current upper limit is $|d_n| < 2.9 \cdot 10^{-26}$ e·cm [1]. If neutron would be enlarged to the size of the Earth it would correspond to a e^+ e^- pair in it's center only few millimeters apart.

Astonishingly it is possible to measure such a small quantity with use of the Larmor precession phenomenon. When an object with a magnetic dipole moment $\vec{\mu}$ is placed in magnetic field $\vec{B}$ it oscillates with Larmor frequency:

$$f_0 = \frac{1}{2\pi\hbar}\vec{\mu} \cdot \vec{B} \tag{1}$$

When electric dipole moment $\vec{d}$ and electric field $\vec{E}$ are also present additional addend of $2\vec{d} \cdot \vec{E}/\hbar$ arises. In simple case two Larmor frequencies occur:

$$\begin{aligned} f_0^+ &= \tfrac{1}{2\pi\hbar}(\mu B + dE) \quad \text{with } \vec{B} \text{ and } \vec{E} \text{ parallel} \\ f_0^- &= \tfrac{1}{2\pi\hbar}(\mu B - dE) \quad \text{with } \vec{B} \text{ and } \vec{E} \text{ antiparallel} \end{aligned} \tag{2}$$

Electric dipole moment can be determined by measuring the difference $\Delta f_0 = f_0^+ - f_0^-$. Method to measure Δf_0 was proposed by Ramsey [2].

The method requiters allowing neutrons to precess freely for as long as possible. To this aim *ultra-cold neutrons* are used, stored in a special storage volume with walls made of material having a large Fermi potential (thus reflecting the ultra-cold neutrons independently on their incident angle).

Ramsey method of separated oscillating fields is incorporated in nEDM experiment at the Paul Scherrer Institute in Villigen, Switzerland [3]. The measurement is carried out in *cycles*, each performed as follows:

1. Polarized ultra-cold neutrons are injected into a storage volume (also called a precession chamber) with spins parallel to the magnetic field and electric field in the volume B_0, E.

2. A pulse of magnetic field perpendicular to B_0, oscillating with frequency close to neutrons' Larmor frequency is applied (called *RF-pulse*). This causes the nutation of neutrons to rotate. Length of the pulse is tuned to rotate their spin direction by $\pi/2$, resulting in neutron spins rotating on a plane perpendicular to B_0.

3. Neutrons are allowed to precess freely for 100-150 s (longer times are not beneficial because of neutron decay and loss by capture by chamber walls).

4. Another $\pi/2$ pulse is applied which is precisely in phase with the first one. Now if neutron precession frequency is exactly equal to one of applied pulses its spin direction would change by another $\pi/2$ resulting in spins anti-parallel to original direction. Any deviation causes a differ-

ence in phase between neutrons oscillation and RF field therefore yielding different change in spin direction. Difference in phase of π yields neutrons spins parallel rather than anti-parallel to B_0. Overall change in spin direction (from both pulses) in function of applied pulses frequency is called *Ramsey resonance curve* (showed in figs. 1 and 2).

5. Orientation of neutrons' spins (neutron polarization) is analyzed: they are dropped through polarization filter and counted.

Each cycle yields a point on Ramsey resonance curve. Then a fit of theoretical curve is performed and actual resonant frequency f_0 obtained.

Together with ultra-cold neutrons the precession chamber is filled with ^{199}Hg atoms which are used to measure magnetic field value averaged over free precession time and chamber volume. These atoms are substantial part of the *mercury co-magnetometer*. Like neutrons, they are polarized and then injected to the precession chamber. A dedicated RF pulse rotates their spins to the horizontal direction. Larmor frequency of their rotation is proportional to the vertical component of the magnetic field B_0 and is measured constantly during free precession time. This is done by measuring absorption of polarized light (emitted by a Hg lamp) shining through the chamber.

Magnetic field in the experiment

Magnetic field plays crucial role in the nEDM experiment, since the effect of nEDM on Larmor frequency is hidden in fluc-

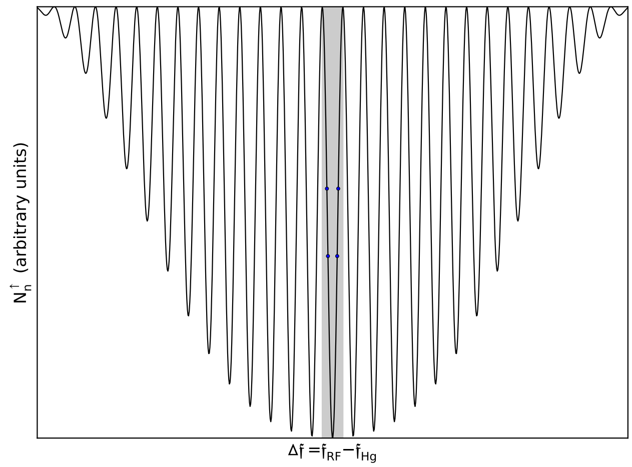

Figure 1: ***Ramsey resonance curve****, i.e. number of counted neutrons with spins in "up" direction (assuming they were polarized "up" at first) after second RF-pulse in function of it's frequency. Fast changing component corresponds to phase difference between neutrons precession and second RF-pulse; the envelope shape is caused by reduced efficiency of RF-pluses in rotating spins. Close-up of grayed area is showed in fig. 2.*

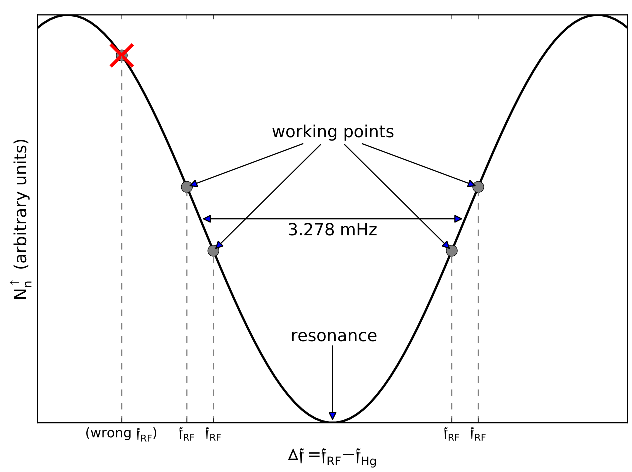

Figure 2: *Close-up of* ***Ramsey resonance curve*** *near resonance. In experiment this curve is probed in* working points *– for each RF-pulse frequency* f_{RF} *neutrons which pass polarization filter are counted* ($N_n^{\uparrow}$).

tuations caused by small changes in ambient magnetic field. These can come from countless sources: Earth's magnetic field, electromagnets running in the institute, computer power supplies, light bulbs. Events such as opening a door are clearly visible in readouts.

Great effort is thus put into measuring and stabilizing the ambient magnetic field. In building hosting the setup number of fluxgates monitor the field and based on their readout gigantic coils winded around the building actively try to stabilize field inside. Four shields made of μ-metal (material with very high magnetic permeability), each of them reducing field by factor of ten, cover the experimental setup itself. Right on top and below the precession chamber sixteen magnetometers are installed. Their readings are used to homogenize field with 30 of so called trim coils. Finally there is already mentioned *mercury co-magnetometer* – mercury atoms precessing together with neutrons in the same volume.

Mercury co-magnetometer is especially important, because it provides the most accurate information about magnetic field neutrons actually feel. Larmor frequency of mercury atoms (f_{Hg}) is measured on-line and used to calculate RF-pulse frequency used in the next measurement cycle. It also allows for *passive compensation* – correcting results for magnetic field fluctuations. Rather than considering the resonance curve as function of pulse frequency f_{RF}, it can be considered a function of detuning frequency $\Delta f := f_{RF} - f_{Hg}$.

RF-PULSE FREQUENCY CHOICE

The experiment probes the Ramsey resonance curve and where the curve is probed is determined by RF-pulse frequency f_{RF}.

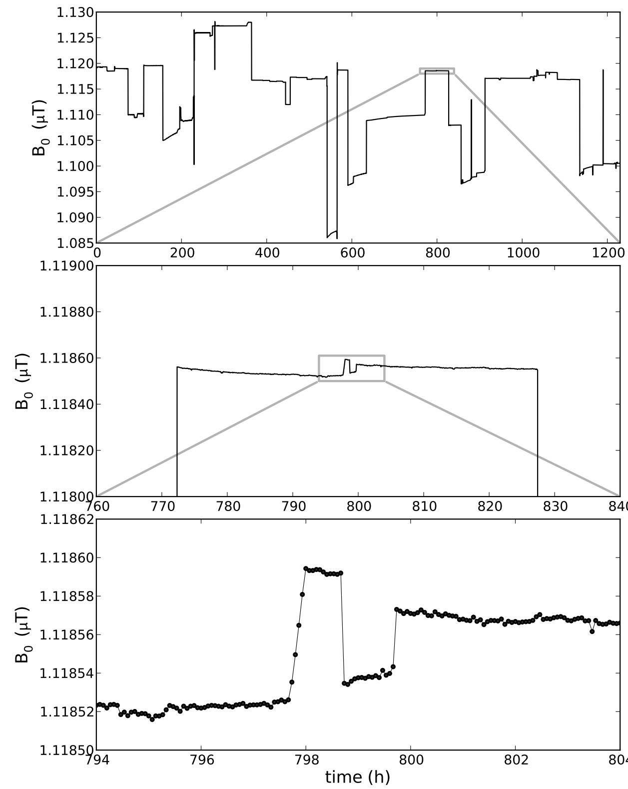

Figure 3: *Typical magnetic field changes measured inside the precession chamber as calculated from f_{Hg}. Each point corresponds to one cycle.*

Then f_0 is determined by fitting theoretical curve.

It turns out that precision of this fit depends on location of data points: those located on steeper slopes are more "valuable". This was the main reason that it was decided to perform measurements in four *working points* (see fig. 2).

Yet the curve is a function of Δf, which can be calculated only when f_{Hg} is known, i.e. after the cycle. This means that when pulse frequency has to be chosen it is not yet known what fragment of resonance curve will be probed. The only way to fix data in working points is to guess f_{Hg}, which is equivalent to need of *predicting the magnetic field for the next cycle*.

Extrapolation of the magnetic field

Magnetic field typically occurring inside the chamber is showed on fig. 3. It's characteristics consists of three major properties:

- **long, smooth changes with characteristic time of several hours** – probably caused by the night–day temperature changes and various activities in the vicinity of the apparatus
- **rapid small-amplitude noise** of instrumental or natural origin
- **sudden jumps in magnetic field intensity** caused mainly by switching outside devices or by mechanical shocks experienced by the magnetic shield

Since properties of the noise are unsure I assumed that it has random character and therefore cannot be predicted. On the other hand, long, smooth changes are inherently predictable.

So what is required is an algorithm for extrapolation of magnetic field in next cycle that will:

- use all available information to make predictions be as accurate as possible
- ignore the noise (and thus make stable predictions)
- quickly recover after sudden jumps

The Model

A simple computer model of the experiment was created in order to investigate various method of choosing RF pulse frequency in the experiment.

In the model function $B_0(t)$ has to be provided – this defines the environment in which the simulated cycles run. Then for each cycle RF-pulse frequency has to be chosen and neutron count N is calculated from formula describing resonance curve and $B_0(t)$ function. Also the counting uncertainties of $\sqrt{N}$ are applied.

After data is generated from numerous cycles resonance curve is fitted to all collected points.

Several additional aspects are introduced in the program, although they were not used in simulations described in this paper:

- B_0 gradients, which cause neutrons to feel different field than mercury atoms due to shift of their center of mass towards bottom of the chamber (they are much colder than mercury)
- fluctuations in number of neutrons provided by UCN source
- fitting resonance curve *during* measurement to last 8 points to find neutrons' resonant frequency on-line – this may serve as a correction for B_0 gradients

Methods of choosing pulse frequency

Each method has to choose such RF-pulse frequency as to probe the resonance curve in proper place. This is equivalent to predicting B_0 for cycle yet to come. Four methods of B_0 extrapolation based on mercury magnetometer data were tested:

Last cycle

The simplest way is to assume that field doesn't change much between adjacent cycles and use as RF-pulse frequency f_{Hg} from last cycle. Main disadvantage of it is that RF-pulse frequency will exactly reproduce any noise occurring in magnetic field.

Average

With possible benefit of smoothing noise an average over last n cycles was considered.

Unfortunately this method is much more vulnerable to sudden jumps than previous one. Effects of such event still hold for n next cycles. Therefore algorithm before making prediction checks how much was it mistaken in last cycle. If deviation exceeds certain threshold than a jump is assumed to occur and algorithm forgets all points, save the last. All methods except the first behave in such way.

Linear extrapolation

More sophisticated way is to fit a line to last n $\mathfrak{f}_{Hg}$ data. When n is chosen properly it allows to ignore any noise in B_0 as well as to follow its long-range changes.

Akaike Information Criterion

Another tested method involves fitting m-degree polynomial to n last points, m being chosen with use of *Akaike Information Criterion* (AIC).

AIC is a relative measure of goodness of fit of model to data. In general case:

$$AIC = 2k - 2\log(L), \tag{3}$$

where k is number of model's parameters and L is likelihood of it describing the data.

Akaike proved using information theory [4] that given a set of models one with the least AIC is the best description of the data, taking into account that model with many parameters which fits data perfectly may be just as good as one with few parameters and worse fit.

In practice several polynomials are fitted to set of points, with degrees ranging from 0 to about 15. For each fit AIC is calculated according to formula:

$$AIC = \log(\Sigma) + \frac{2(d+1)}{n}, \tag{4}$$

where Σ is sum of squared deviations, d is polynomial's degree. Function with the lowest AIC is used to predict next $\mathfrak{f}_{Hg}$ value.

This method could be capable of not only smoothening noise but also following more complex changes in magnetic field.

Testing and Results

Each method (except the simplest one) has an important parameter – n. All of them were thus tested for values of $n \in [1, 39]$.

For each method and each n a very long run spanning over 840 hours was simulated with real experimental data of $\mathfrak{f}_{Hg}$ used as $B_0(t)$. Each cycle yielded a value $\mathfrak{f}_{mistake} = \mathfrak{f}_{guess} - \mathfrak{f}_{Hg}$, which distribution was than analyzed. It's standard deviation for each method is plotted in function of n on fig. 4.

As can be seen on the plot, optimal values of n are: 2 for averaging, 11 for linear extrapolation and 17 for AIC extrapolation. Data generated with all methods set to their optimal n is shown on fig. 5.

Introducing an offset

Any method for choosing RF-pulse frequency may only improve fixation of data in working points, which is not by itself valuable. Working points were introduced to improve accuracy of the final fit, so a

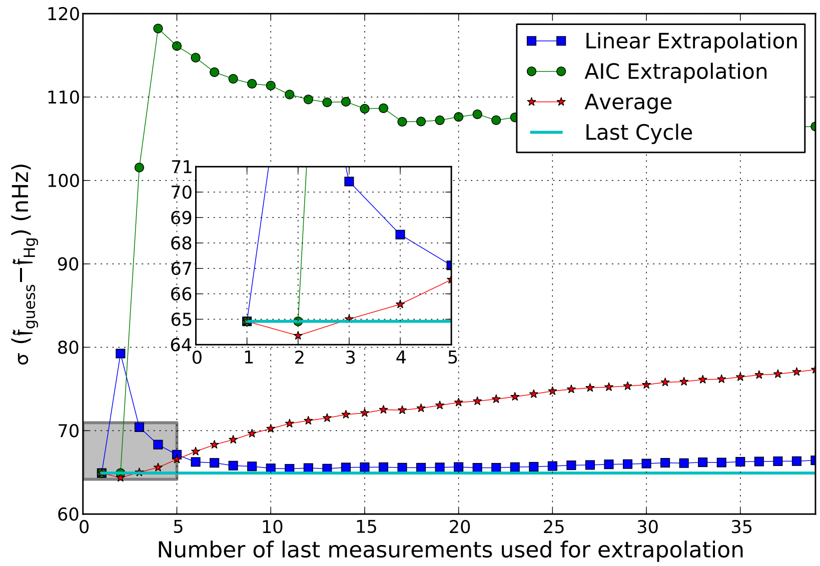

Figure 4: *Standard deviation of* $f_{mistake} = f_{guess} - f_{Hg}$ *plotted in function of* n *– number of last cycles from which* f_{Hg} *was used for extrapolation.*

question arises: *does improving fixation of data in working points improve accuracy of final fit?*

To investigate that I decided to compare influence of introducing *an offset* in location of working points (see fig. 6) with possible influence of choosing one method over another. There are few parts of the experiment where systematic effects may cause such offset:

- B_0 gradients inside precession chamber
- fitting mercury magnetometer signal to determine f_{Hg}
- RF-pulse generator

As a reference *Nostradamus* was introduced – a method which always predicts correctly and thus sets all data precisely in working points. Then for offsets ranging between -2 and 2 μHz number of simulations were performed, each yielding uncertainty $\sigma(f_0)$ of final fit for every method. Results are shown on fig. 7.

Table 1: *Uncertainties of final fit to data generated with methods with their optimal* n. *Statistical is error calculated from sample of 100 simulations per method.*

Linear Extr.	$(180.2 \pm 1.5\,\text{stat.})$ pHz
AIC Extr.	$(180.70 \pm 0.93\,\text{stat.})$ pHz
Average	$(180.28 \pm 0.90\,\text{stat.})$ pHz
Last Cycle	$(180.16 \pm 0.81\,\text{stat.})$ pHz
Nostradamus	$(179.48 \pm 0.93\,\text{stat.})$ pHz

Since zero offset is the most interesting statistical sample of simulations were performed for each method to find statistical uncertainty of f_0. Results are shown in tab. 1.

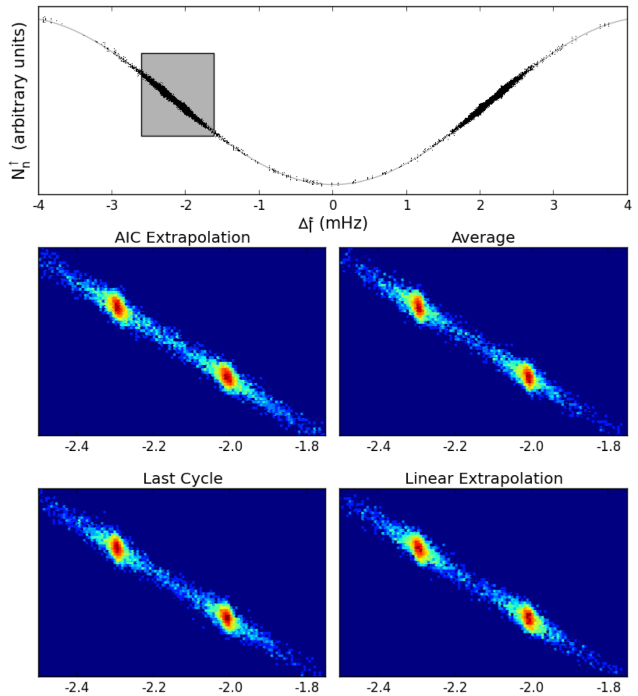

Figure 5: *Data generated with choosing RF-pulse frequency with different methods. Four bottom plots are 2D histograms of area grayed on the top one, one for each RF-pule selection method.*

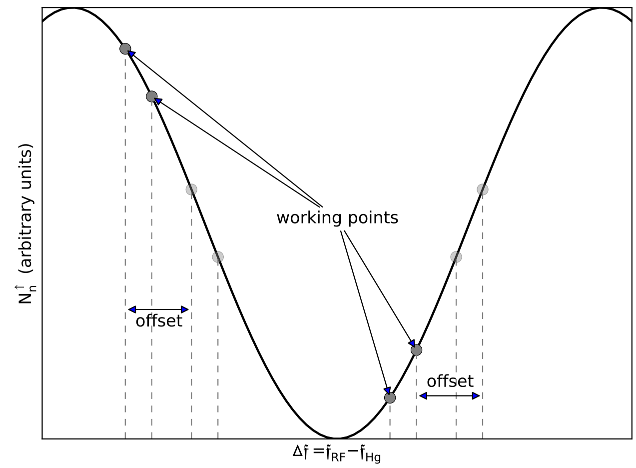

Figure 6: *Graphical explanation of introducing a systematic offset in location of working points.*

Conclusions

It turns out that the best method in predicting B_0 field is averaging last two results. Still the simplest one – using mercury magnetometer result from previous cycle proved to work surprisingly well. This might be explained by following fact: cycle length (≈ 150 s) lies in proximity of the magnetic field's *Allan's standard deviation* minimum. This means that B_0 is the most stable when averaged over times close to cycle length, thus assuming it to be constant in such time-scale is well based.

Even thought data generated with different methods may seem not to differ much (fig. 5), method's performance does reflect in uncertainty of f_0 determined in final fit. The influence's order is several pHz.

Furthermore, working points' location showed to influence uncertainty of neutron resonance location determined in final fit, meaning that any systematic effects causing an offset in their location should be avoided.

Last but not least, all proposed methods provide data fixation in working points sufficient to achieve quality of final fit as good as is achieved with ideal foreseer method.

References

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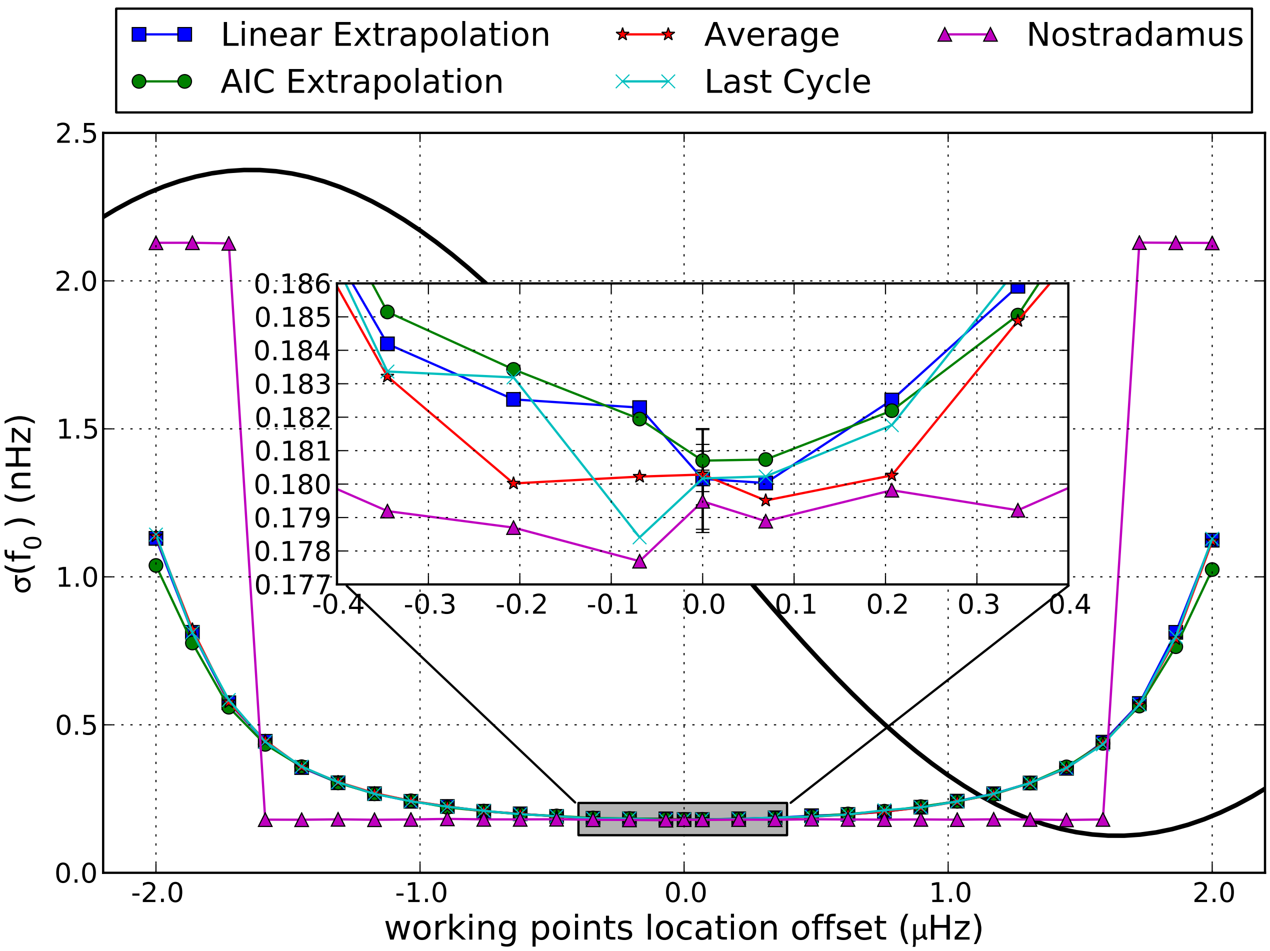

Figure 7: *Influence of an offset in location of working points (see fig. 6) on accuracy of final fit* $\sigma(f_0)$ *for all methods of RF-pulse frequency selection (Nostradamus always predicts correctly). Black resonance curve is drawn for clarity:* x *coordinate of every data point corresponds to center of location of two leftmost working points on resonance curve drawn. Error bars are shown only for offset 0, since only for this data statistical samples were generated.*